

# Neural variability and activity normalization

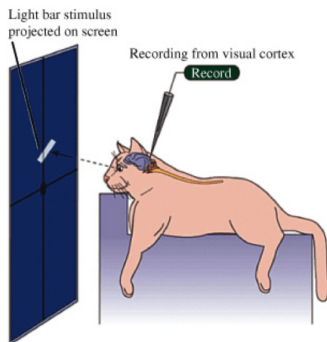
Daniel Herrera-Esposito

Department of Psychology  
University of Pennsylvania



# Introduction: Neural variability

- Neural activity is variable

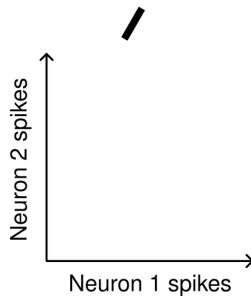


Neuron 1

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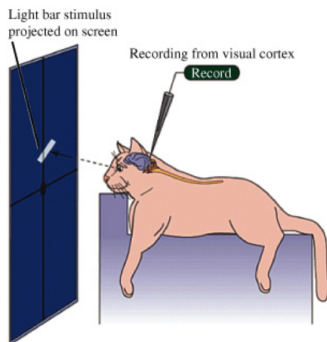
Neuron 2

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# Introduction: Neural variability

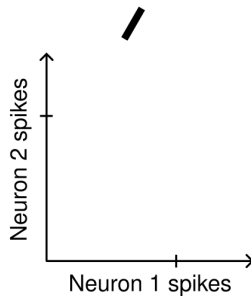
- Neural activity is variable



Neuron 1

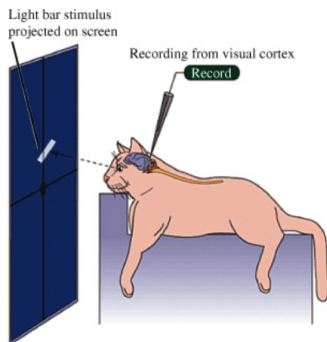


Neuron 2



# Introduction: Neural variability

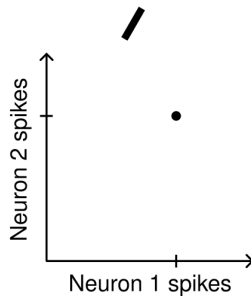
- Neural activity is variable



Neuron 1

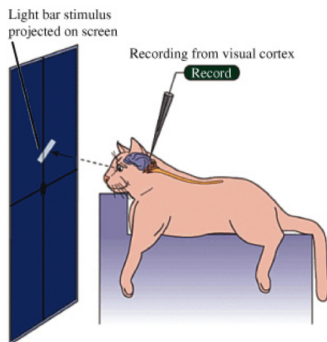


Neuron 2



# Introduction: Neural variability

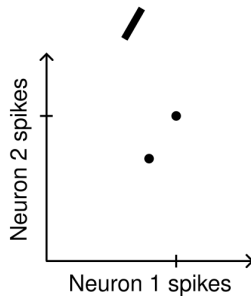
- Neural activity is variable



Neuron 1

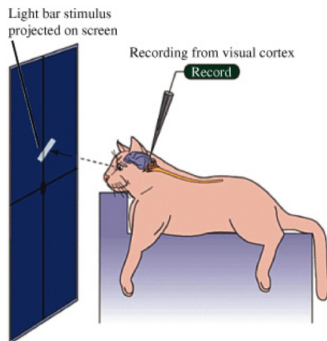


Neuron 2



# Introduction: Neural variability

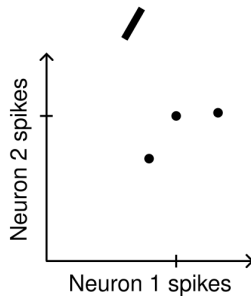
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Neuron 1

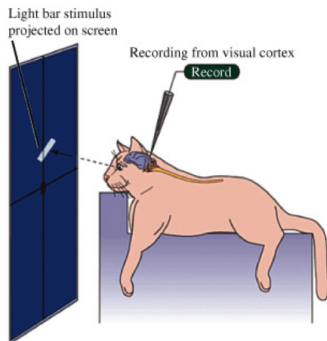


Neuron 2



# Introduction: Neural variability

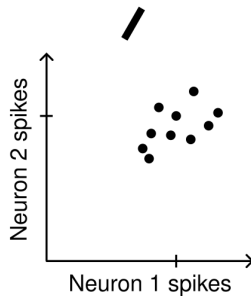
- Neural activity is variable



Neuron 1

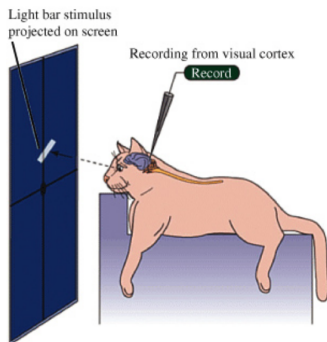


Neuron 2



# Introduction: Neural variability

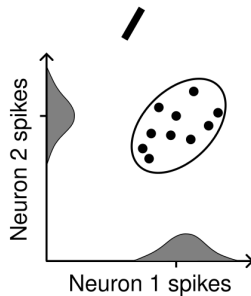
- Neural activity is variable



Neuron 1



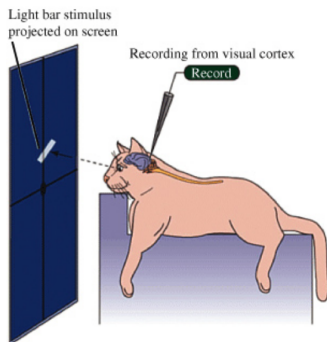
Neuron 2





# Introduction: Neural variability

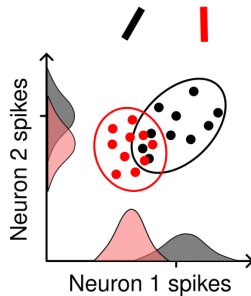
- Neural activity is variable
- Variability is flexible



Neuron 1

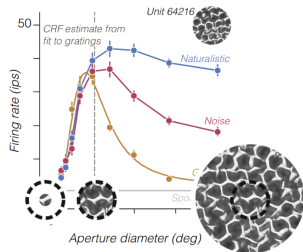
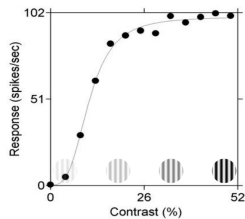


Neuron 2



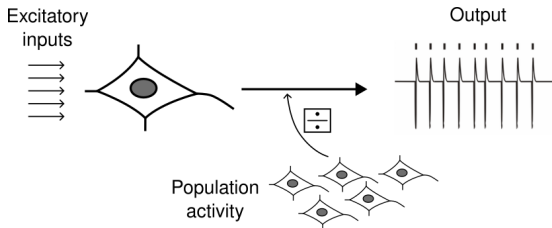
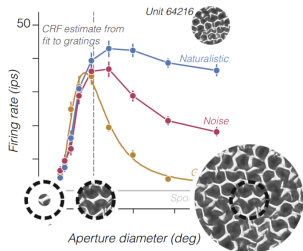
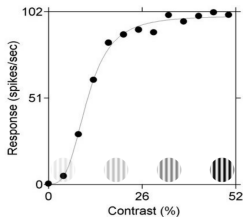
# Introduction: Divisive normalization

- Neural activity is often sub-linear



# Introduction: Divisive normalization

- Neural activity is often sub-linear
- A model of this is Divisive Normalization



$$y = \frac{x}{\sqrt{x' B x}}$$

# Divisive normalization and neural variability interact

- Normalization-variability interaction is important to understand neural coding

nature reviews neuroscience

<https://doi.org/10.1038/s41583-024-00795-0>

Perspective

 Check for updates

## Response sub-additivity and variability quenching in visual cortex

Robbe L. T. Goris<sup>1</sup>✉, Ruben Coen-Cagli<sup>2,3,4</sup>, Kenneth D. Miller<sup>5,6,7,8,9</sup>, Nicholas J. Priebe<sup>10</sup> & Máté Lengyel<sup>11,12</sup>

### Attention-related changes in correlated neuronal activity arise from normalization mechanisms

Bram-Ernst Verhoef<sup>1,2</sup>✉ & John H R Maunsell<sup>1</sup>✉

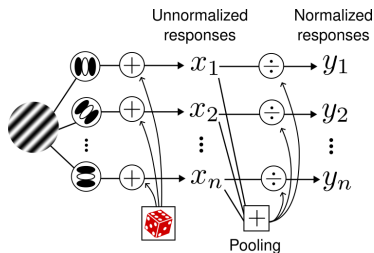
# Our model

Our model:

- Classical normalization model
- Allow variability

$$\mathbf{y} = \frac{\mathbf{x}}{\sqrt{\mathbf{x}' \mathbf{B} \mathbf{x}}}$$

$$\mathbf{x} \sim \mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$$



# Our model

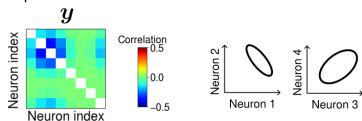
What we did:

- We derive formulas for the normalized response statistics

$$\mathbf{y} = \frac{\mathbf{x}}{\sqrt{\mathbf{x}' \mathbf{B} \mathbf{x}}}$$

$$\mathbf{x} \sim \mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$$

Output correlation matrix



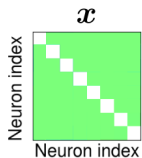
$$\mathbb{E}[y_i] = \mathbb{E} \left[ \frac{x_i}{\sqrt{x_i^2 B_{ii} + v_i}} \right] \approx \frac{\mu_i}{\sqrt{\mu_i^2 B_{ii} + \hat{v}_i}} + \text{tr} \left( \mathbf{H}_{y_i}(\mu_i, \hat{v}_i) \cdot \begin{bmatrix} \sigma_{x_i}^2 & \rho_{x_i, v_i} \\ \rho_{x_i, v_i} & \sigma_{v_i}^2 \end{bmatrix} \right)$$

$$\mathbb{E}[y_i y_j] = \mathbb{E} \left[ \frac{x_i x_j}{\mathbf{x}^T \mathbf{B} \mathbf{x}} \right] = \mathbb{E} \left[ \frac{N}{D} \right] \approx \frac{\mu_N}{\mu_D} \left( 1 - \frac{\rho(N, D)}{\mu_N \mu_D} + \frac{\sigma_D^2}{\mu_D^2} \right)$$

# Results

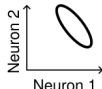
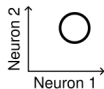
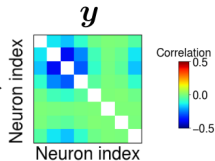
- Our model produces known phenomena and new insights:
  - **Normalization-induced correlations**
  - Stimulus dependent correlations
  - Correlation structure follows normalization weights

Input correlations

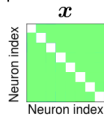


Normalization

Output correlations

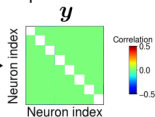


Input correlations



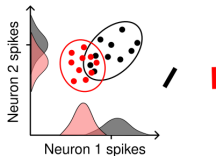
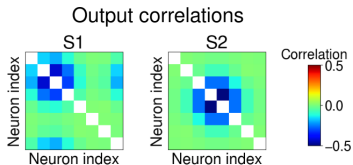
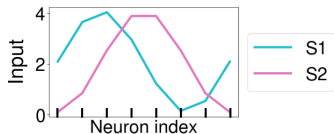
No  
Normalization

Output correlations



# Results

- Our model produces known phenomena and new insights:
  - Normalization-induced correlations
  - **Stimulus dependent correlations**
  - Correlation structure follows normalization weights

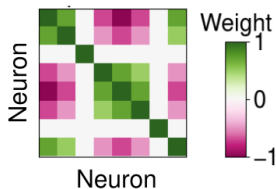




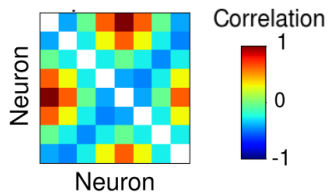
# Results

- Our model produces known phenomena and new insights:
  - Normalization-induced correlations
  - Stimulus dependent correlations
  - **Correlation structure follows normalization weights**

Normalization weights



Correlations



# Next steps

- Test our model with classic neural variability-experiments
- Use our analytic formulas to fit neural data

# Some amazing tools that helped me

- How does a biologist get all this math done?
- Online math forums

# Some amazing tools that helped me

## Math forums:

The screenshot shows a Stack Exchange question on the Cross Validated site. The question title is "Formulas, approximations, or bounds for  $\mathbb{E} \left( \frac{X}{\|X\|} \right)$ ,  $X \sim N(\mu, \Sigma)$ ?" It was asked 1 year ago, modified 11 months ago, and viewed 409 times. The question body contains a reference to another question and a request for a formula. A comment by user225256 provides a detailed answer, stating that the expected value  $E[X/\|X\|]$  is of the form  $f(U, V) = U/\sqrt{U^2 + V}$ , where  $U$  is the component of  $X$  in the direction of  $\mu$  and  $V$  is the squared norm of the component orthogonal to  $\mu$ . The answer suggests approximating  $f$  with a second-order Taylor series around  $(E[U], E[V])$ .

## Expected value of Rayleigh quotient, non-centered Gaussian vector

Asked 1 year, 8 months ago Modified 1 year, 8 months ago Viewed 366 times

Formulas or approximations for  $\mathbb{E} \left( \frac{X}{\|X\|} \right)$  or  $\mathbb{E} \left( \frac{X}{\|X\|^2} \right)$ ,  $X \sim N(\mu, Id)$ ?

Asked 1 year ago Modified 6 months ago Viewed 391 times

Ask Question

## Prove an inequality with confluent hypergeometric functions

Asked 11 months ago Modified 11 months ago Viewed 39 times

Ask Question

# Some amazing tools that helped me

## Math forums:

- Using math/programming/statistics forums takes work
- They can help you at any level of knowledge
- Experts around the world happy to help

# Conclusion

- Neural variability and divisive normalization interact
- We have a model that captures this interaction and provides us with insight
- The model can be fit to data and used to make predictions

Thanks:

Thanks:



jmarkov

784  4  11



whuber 

332k  63  786  1.3k



mathlove

146k  9  121  297






Ben

854  4  4

- user225256



Roland F

3,914  1  2  13



whpowell96

7,408  1  11  23