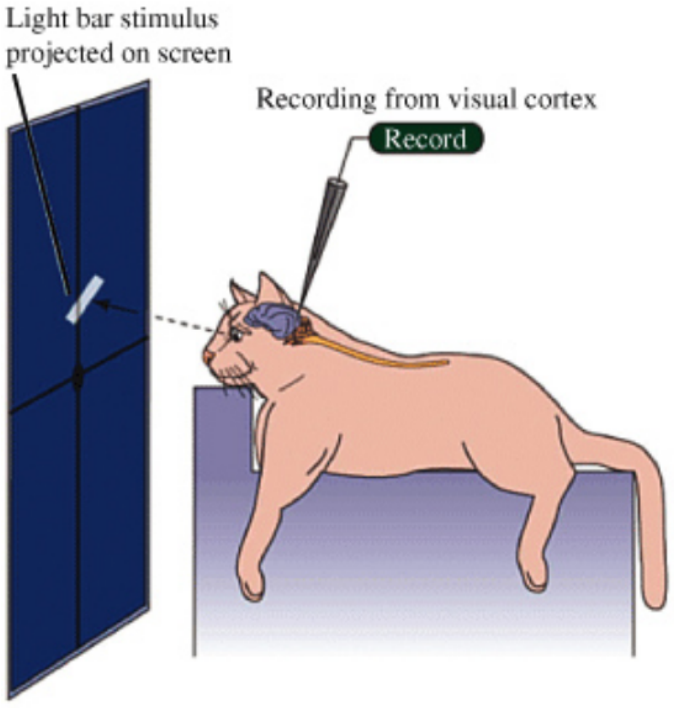
Neural variability and activity normalization

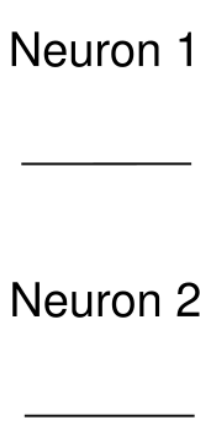
Daniel Herrera-Esposito

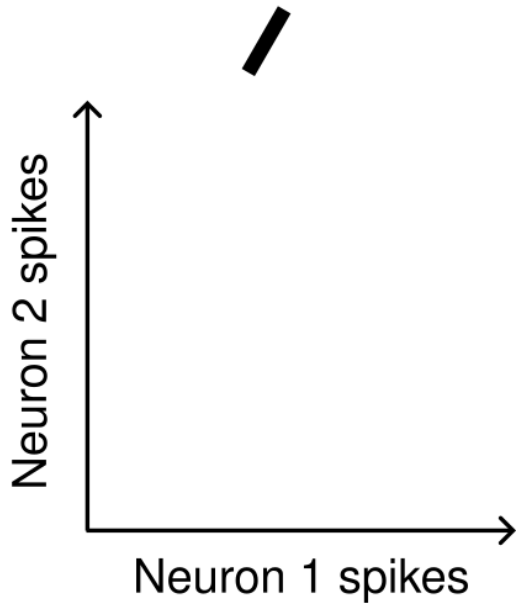
Department of Psychology
University of Pennsylvania

Introduction: Neural variability

- Neural activity is variable

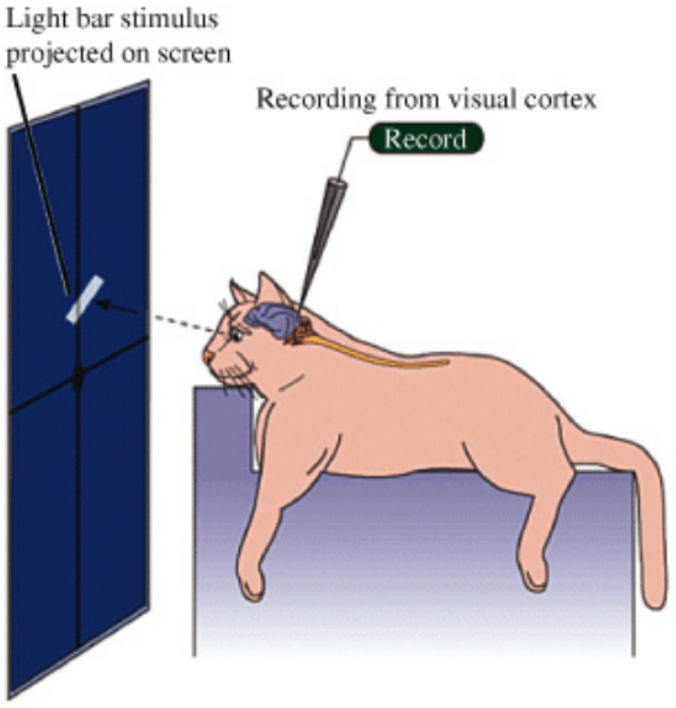


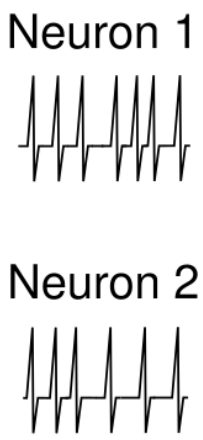


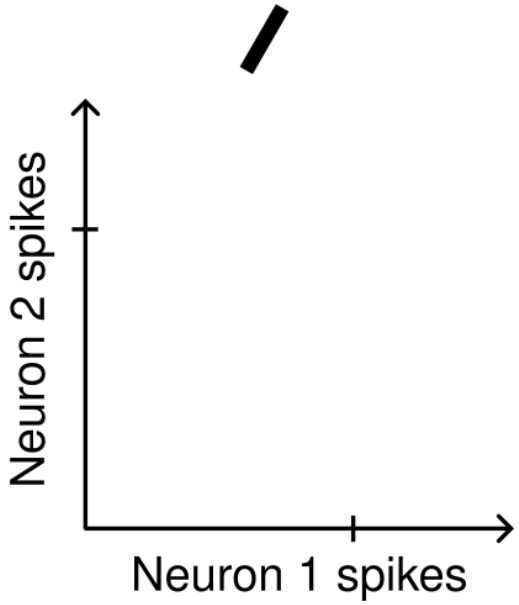


Introduction: Neural variability

- Neural activity is variable

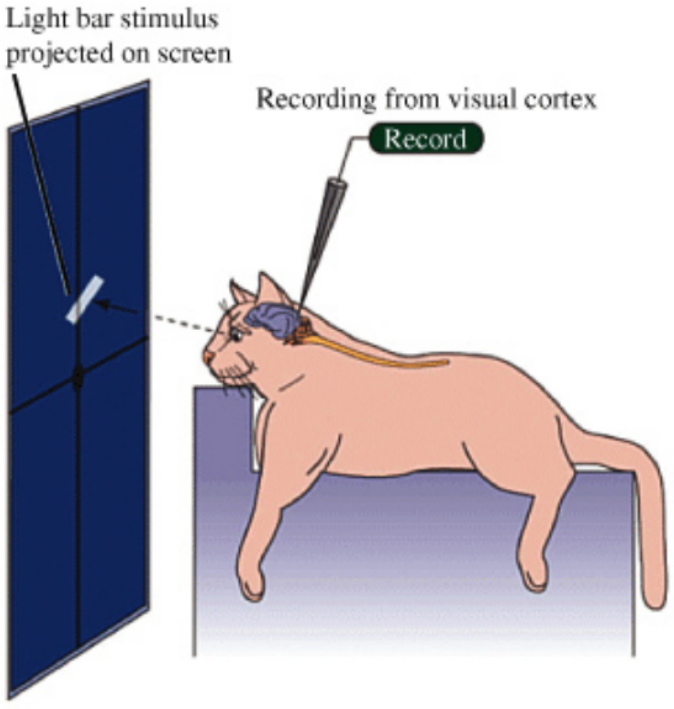


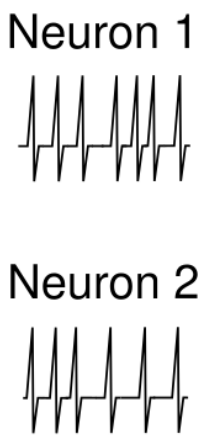


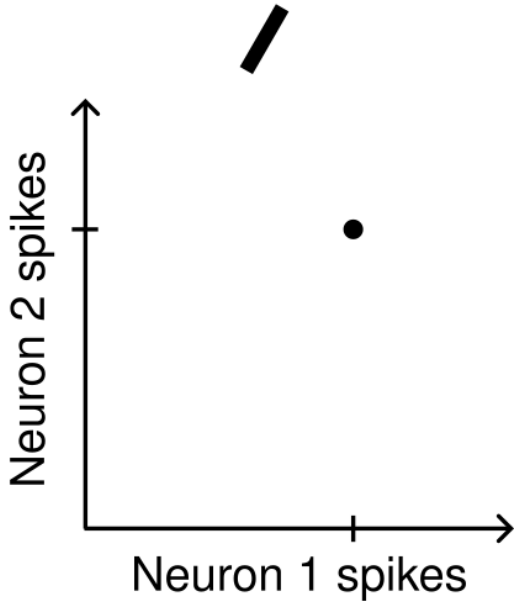


Introduction: Neural variability

- Neural activity is variable

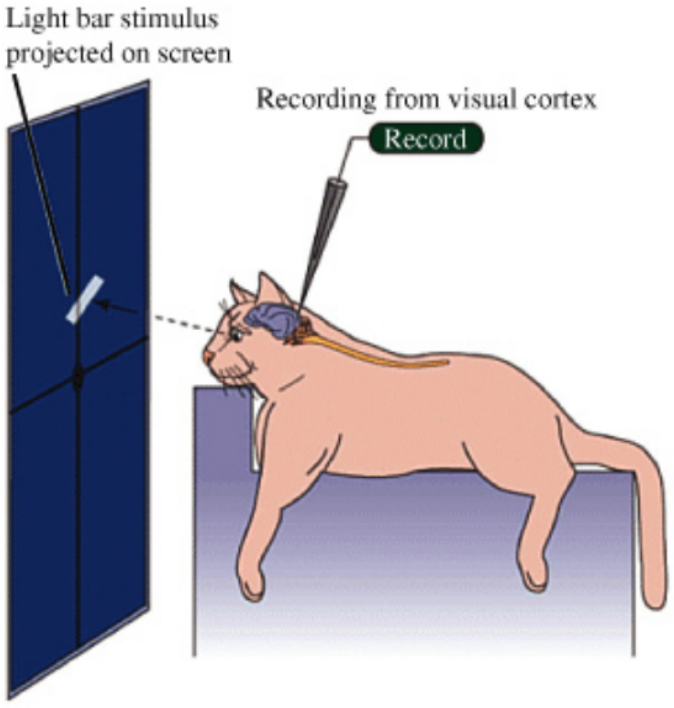


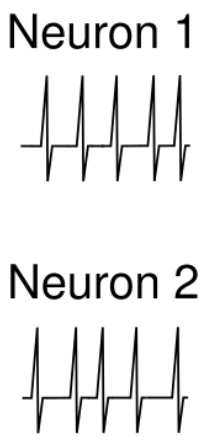


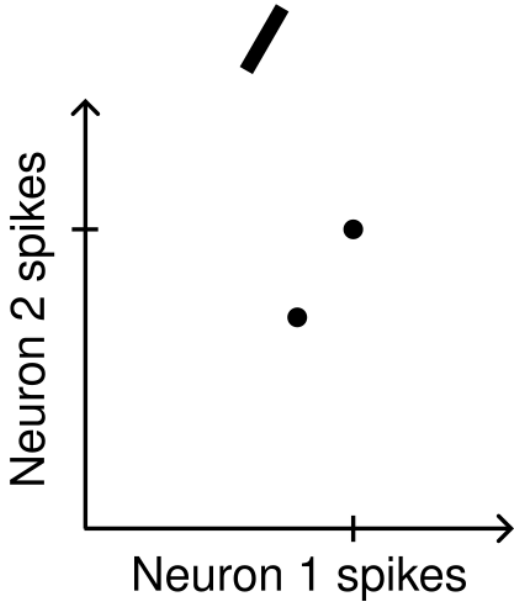


Introduction: Neural variability

- Neural activity is variable

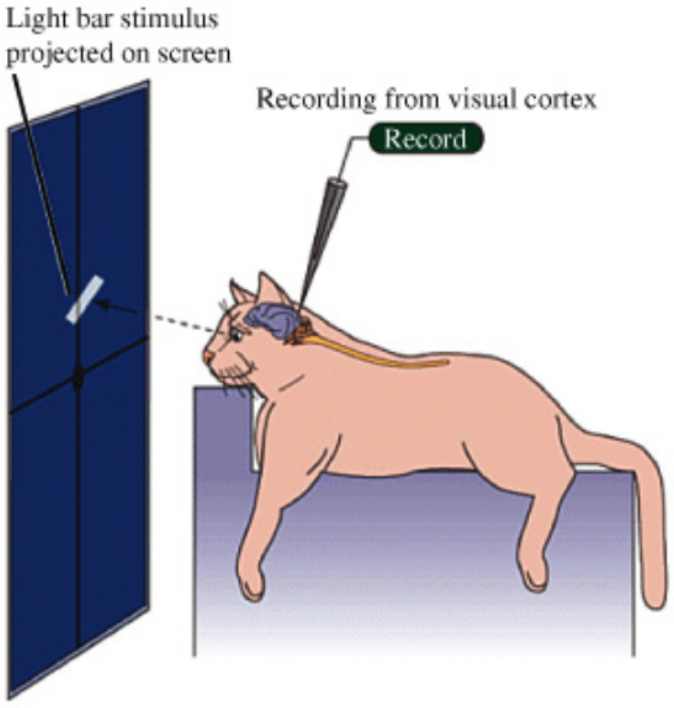


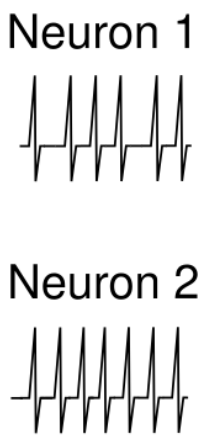


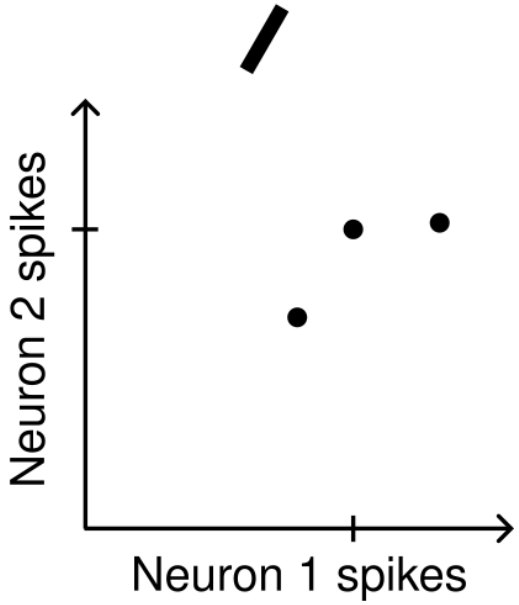


Introduction: Neural variability

- Neural activity is variable

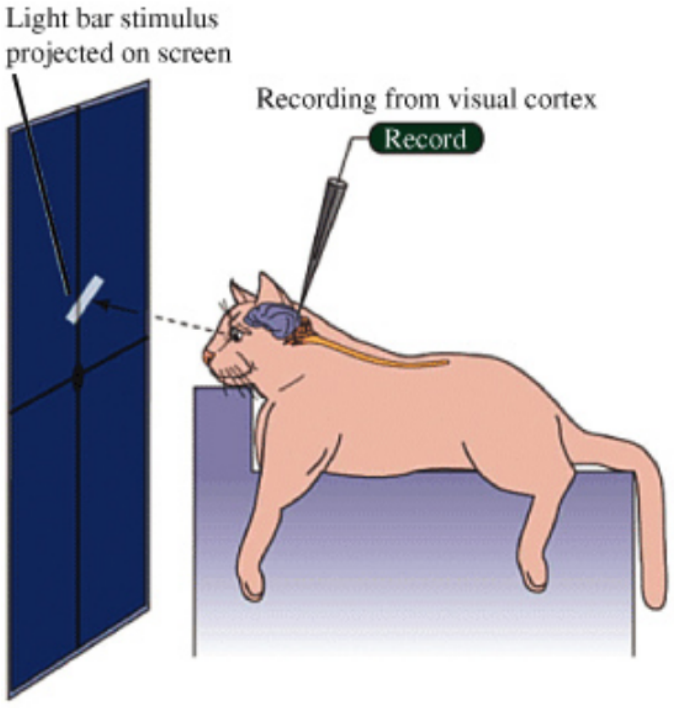


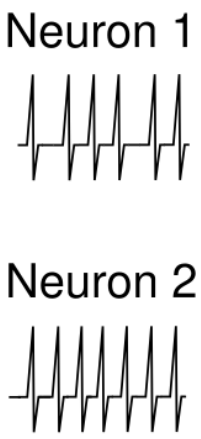


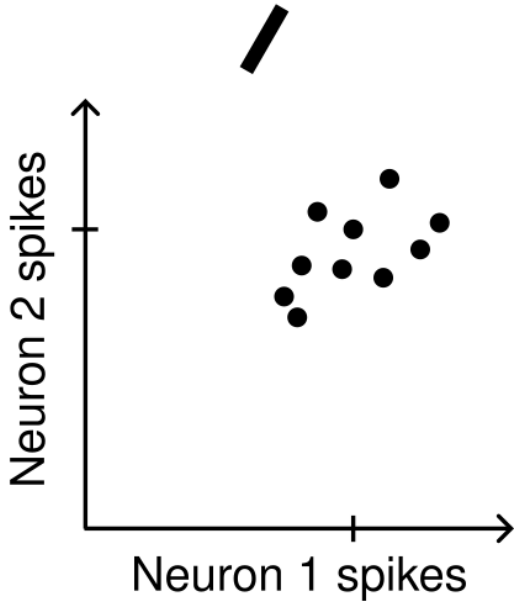


Introduction: Neural variability

- Neural activity is variable

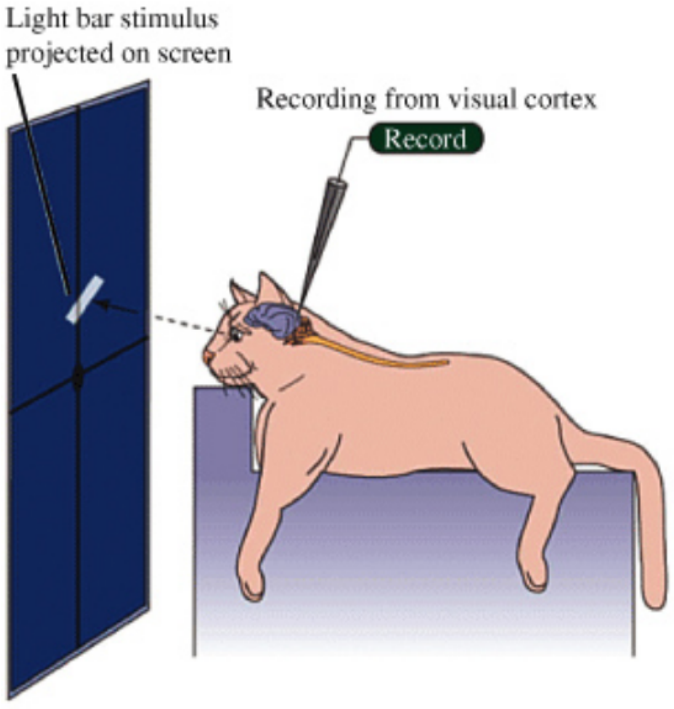


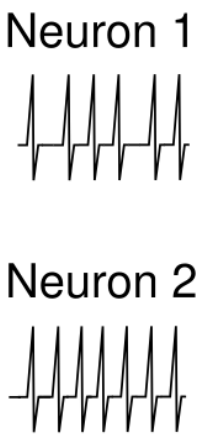


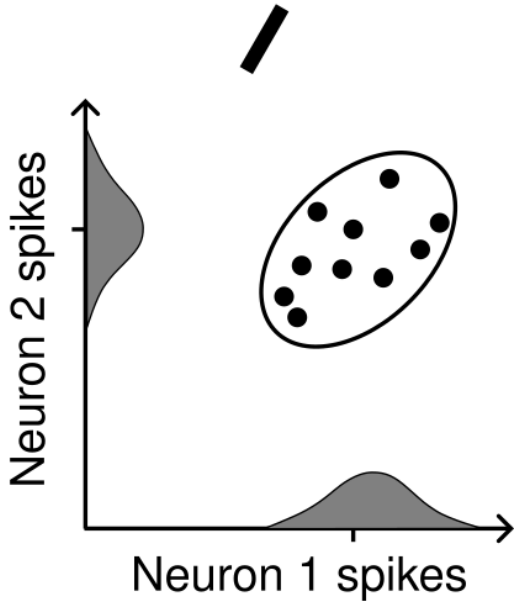


Introduction: Neural variability

- Neural activity is variable

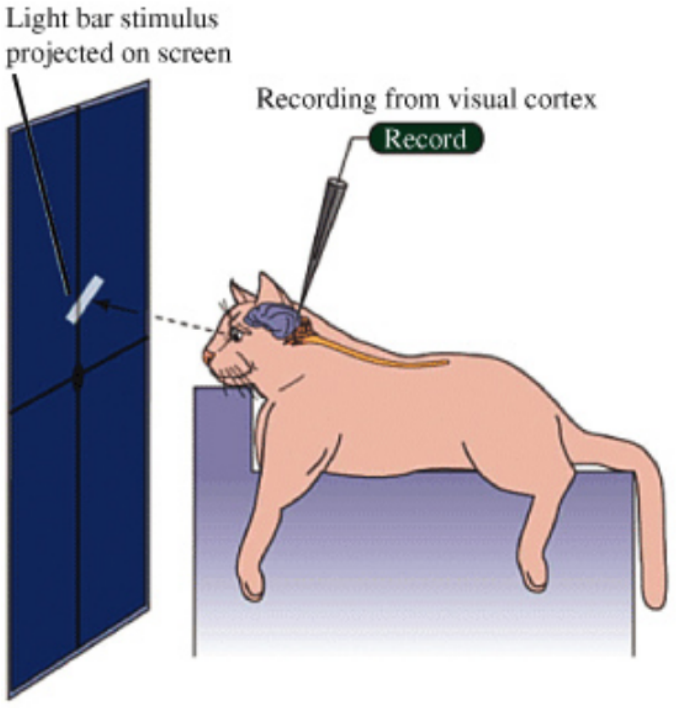


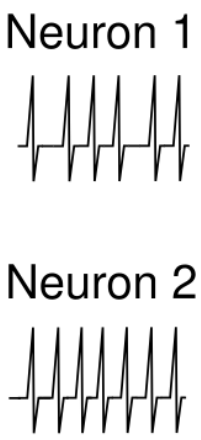


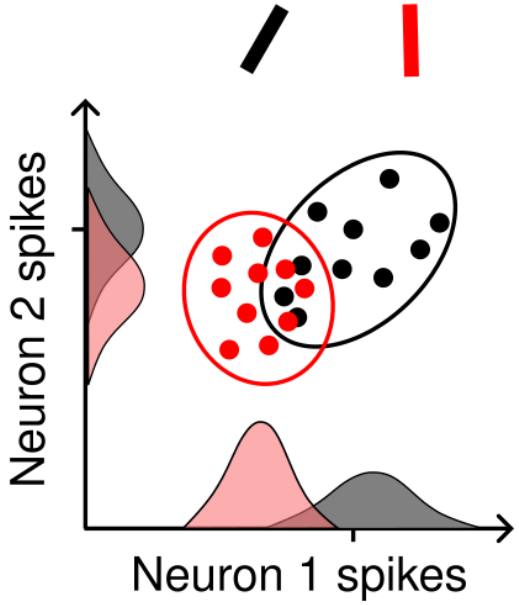


Introduction: Neural variability

- Neural activity is variable
- Variability is flexible

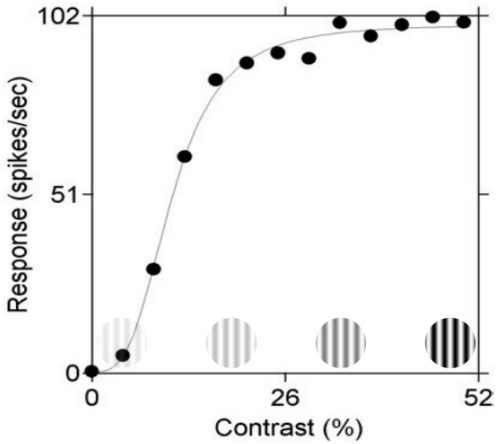


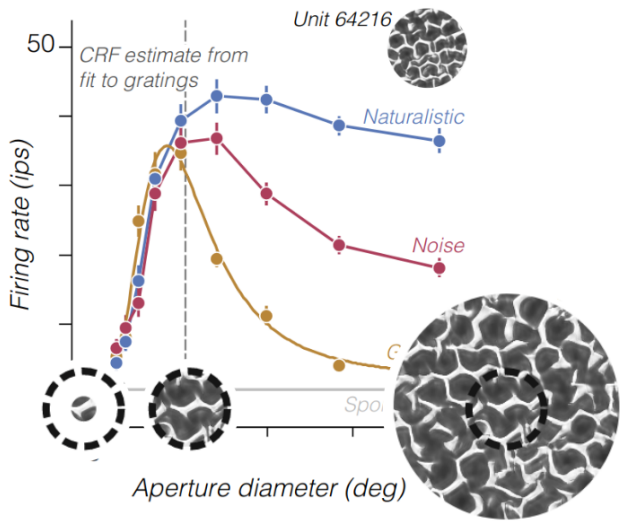




Introduction: Divisive normalization

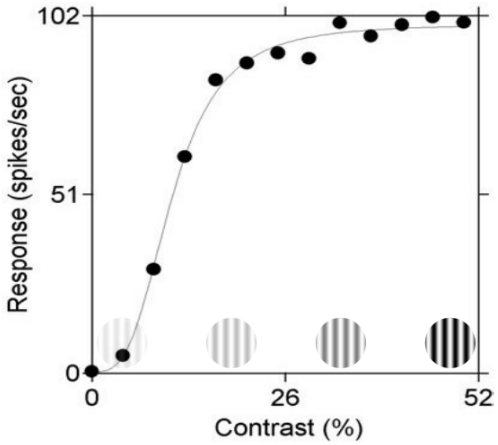
- Neural activity is often sub-linear

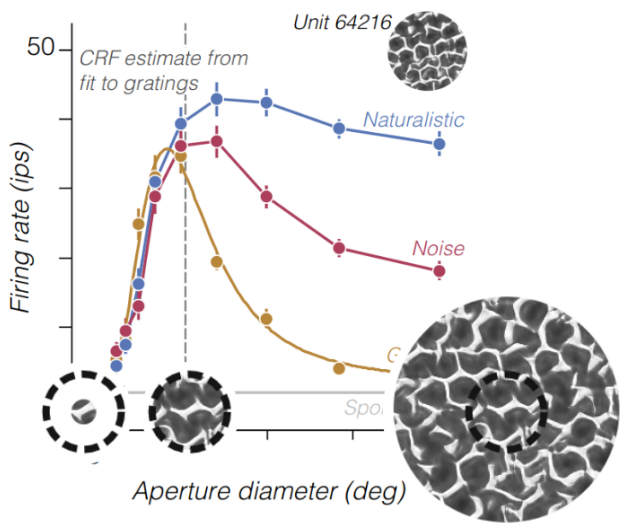


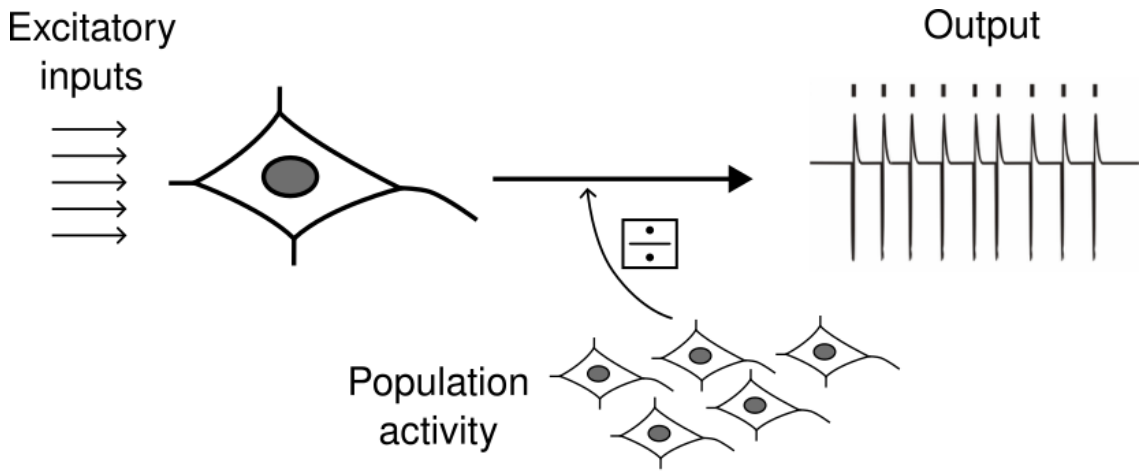


Introduction: Divisive normalization

- Neural activity is often sub-linear
- A model of this is Divisive Normalization







$$\mathbf{y} = \frac{\mathbf{x}}{\sqrt{\mathbf{x}'\mathbf{B}\mathbf{x}}}$$

Divisive normalization and neural variability interact

- Normalization-variability interaction is important to understand neural coding

nature reviews neuroscience

https://doi.org/10.1038/s41583-024-00795-0

Perspective

Check for updates

Response sub-additivity and variability quenching in visual cortex

Robbe L. T. Goris[1], Ruben Coen-Cagli[2,3,4], Kenneth D. Miller[5,6,7,8,9], Nicholas J. Priebe[10] & Máté Lengyel[11,12]

Attention-related changes in correlated neuronal activity arise from normalization mechanisms

Bram-Ernst Verhoef[1,2] & John H R Maunsell[1]

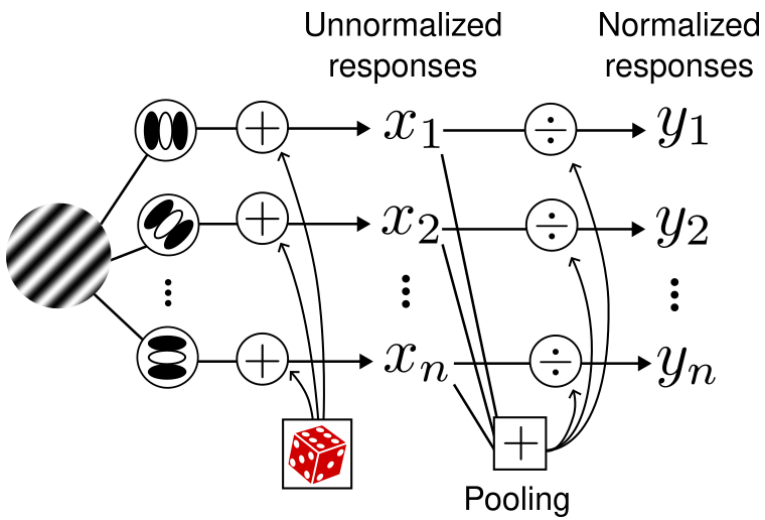
Our model

Our model:

- Classical normalization model
- Allow variability

$$\boldsymbol{y} = \frac{\boldsymbol{x}}{\sqrt{\boldsymbol{x}' \boldsymbol{B} \boldsymbol{x}}}$$

$$\boldsymbol{x} \sim \mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$$



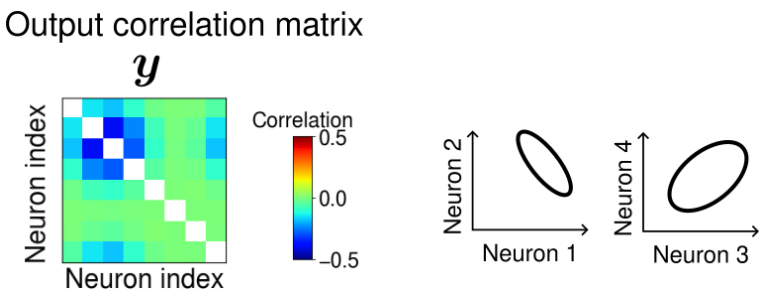
Our model

What we did:

- We derive formulas for the normalized response statistics

$$\boldsymbol{y} = \frac{\boldsymbol{x}}{\sqrt{\boldsymbol{x}' \boldsymbol{B} \boldsymbol{x}}}$$

$$\boldsymbol{x} \sim \mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$$

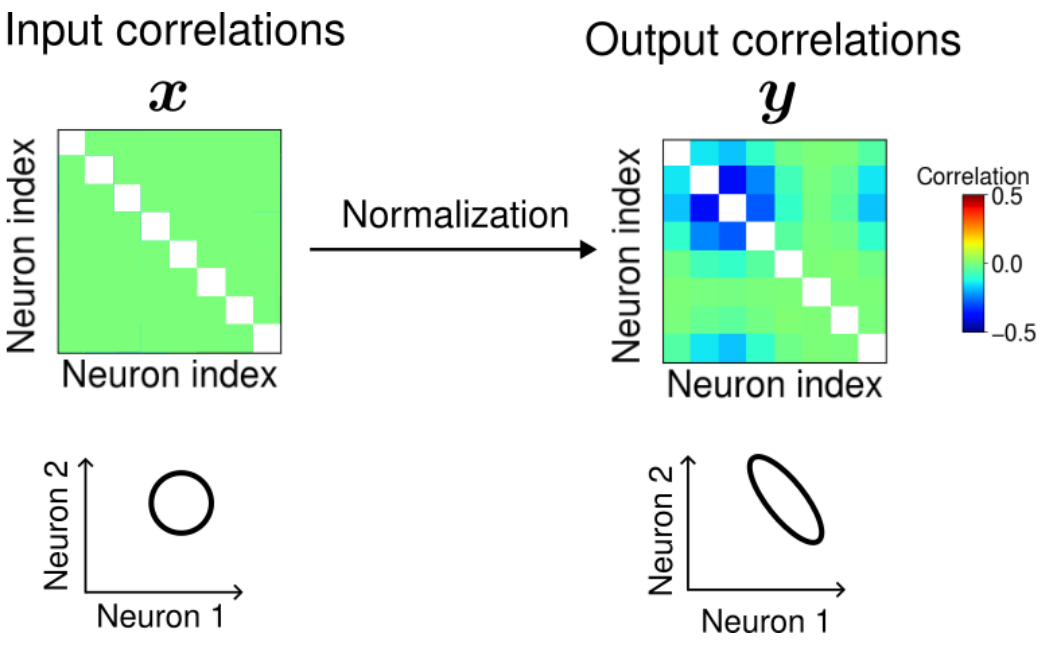


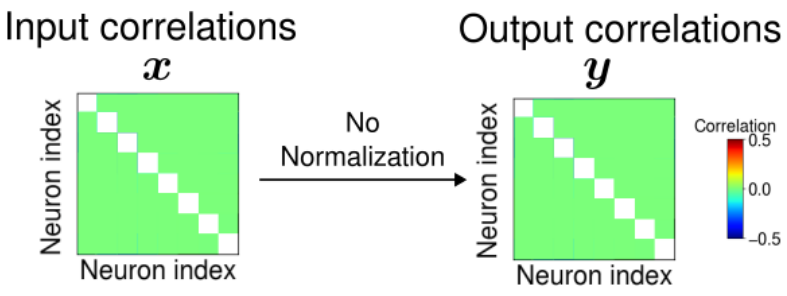
$$\mathbb{E}[y_i] = \mathbb{E}\left[\frac{x_i}{\sqrt{x_i^2 B_{ii} + v_i}}\right] \approx \frac{\mu_i}{\sqrt{\mu_i^2 B_{ii} + \widehat{v_i}}} + \operatorname{tr}\left(\mathbf{H}_{y_i}(\mu_i, \widehat{v_i}) \cdot \begin{bmatrix} \sigma_{x_i}^2 & \rho_{x_i,v_i} \\ \rho_{x_i,v_i} & \sigma_{v_i}^2 \end{bmatrix}\right)$$

$$\mathbb{E}[y_i y_j] = \mathbb{E}\left[\frac{x_i x_j}{\mathbf{x}^T \mathbf{B} \mathbf{x}}\right] = \mathbb{E}\left[\frac{N}{D}\right] \approx \frac{\mu_N}{\mu_D}\left(1 - \frac{\rho(N,D)}{\mu_N \mu_D} + \frac{\sigma_D^2}{\mu_D^2}\right)$$

Results

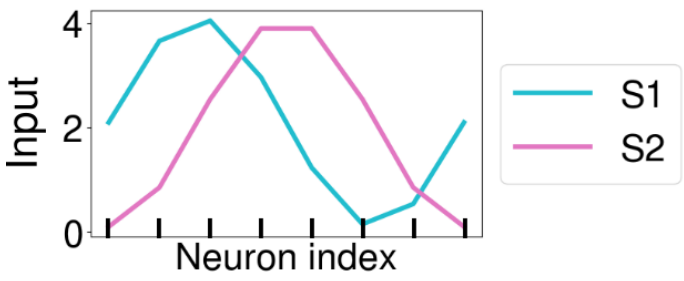
- Our model produces known phenomena and new insights:
  - Normalization-induced correlations
  - Stimulus dependent correlations
  - Correlation structure follows normalization weights

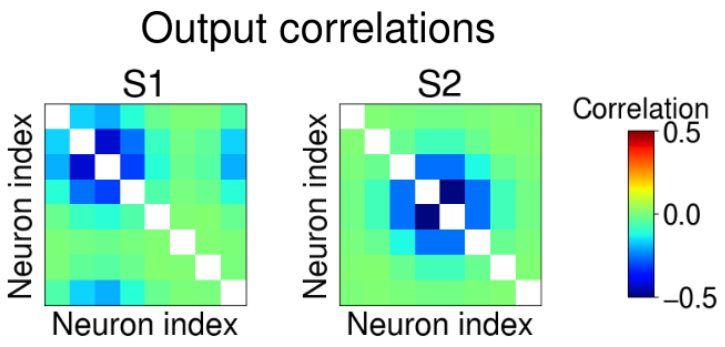


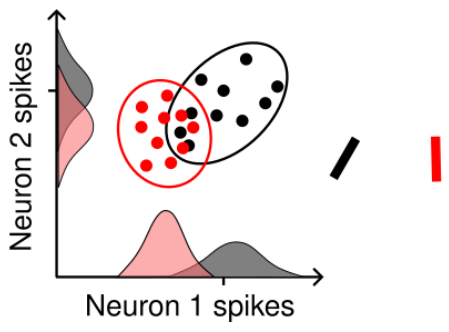


Results

- Our model produces known phenomena and new insights:
  - Normalization-induced correlations
  - Stimulus dependent correlations
  - Correlation structure follows normalization weights

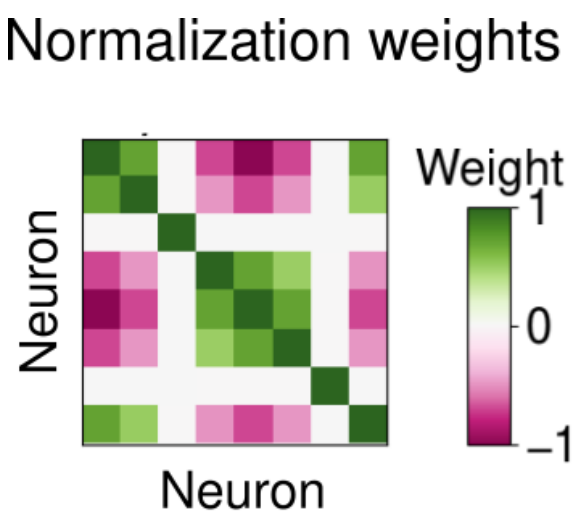


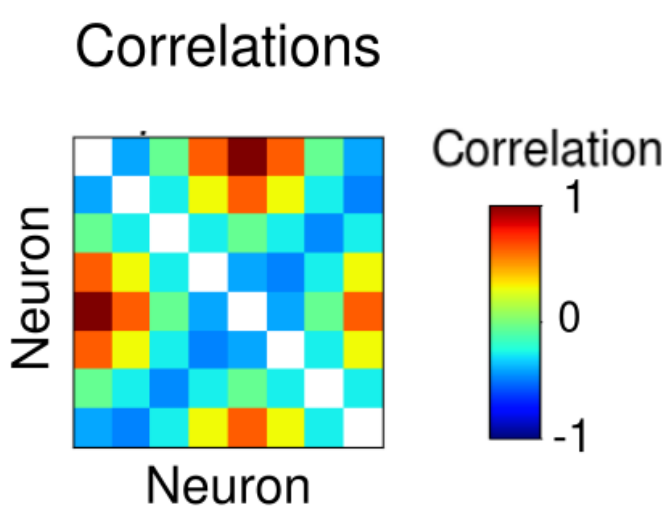




Results

- Our model produces known phenomena and new insights:
  - Normalization-induced correlations
  - Stimulus dependent correlations
  - Correlation structure follows normalization weights





Next steps

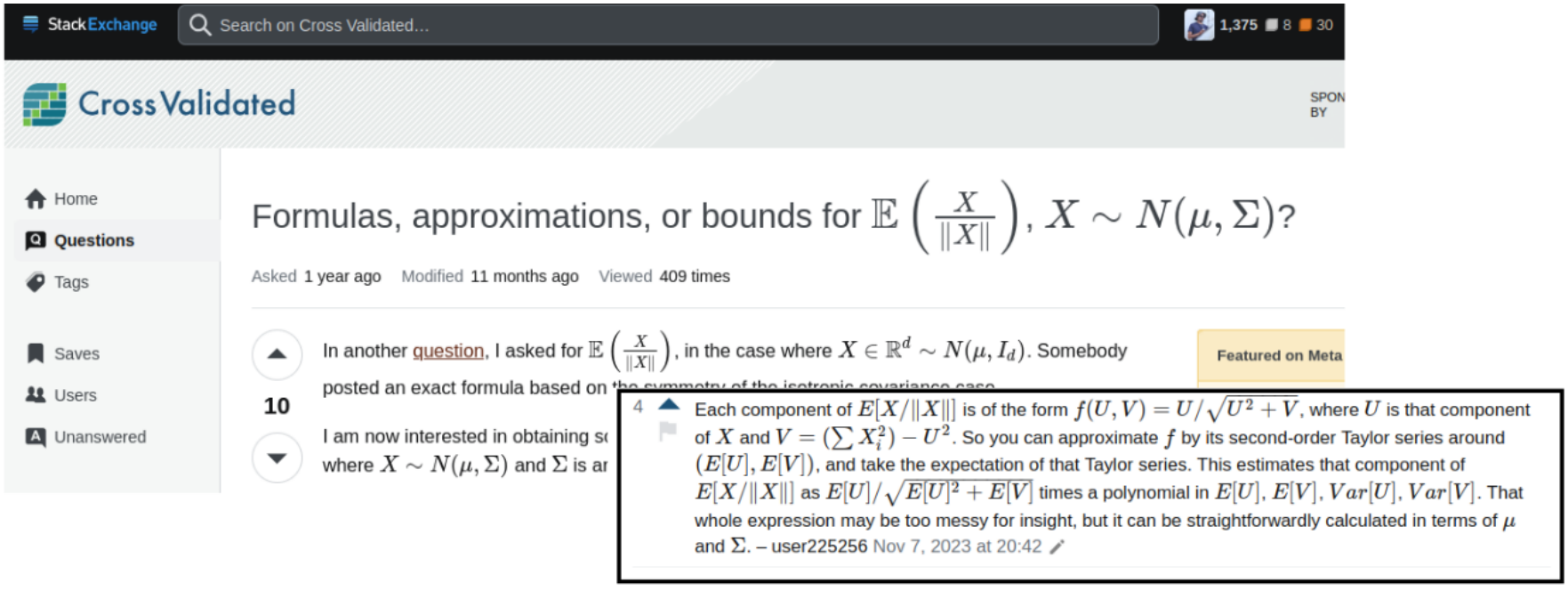
- Test our model with classic neural variability-experiments
- Use our analytic formulas to fit neural data

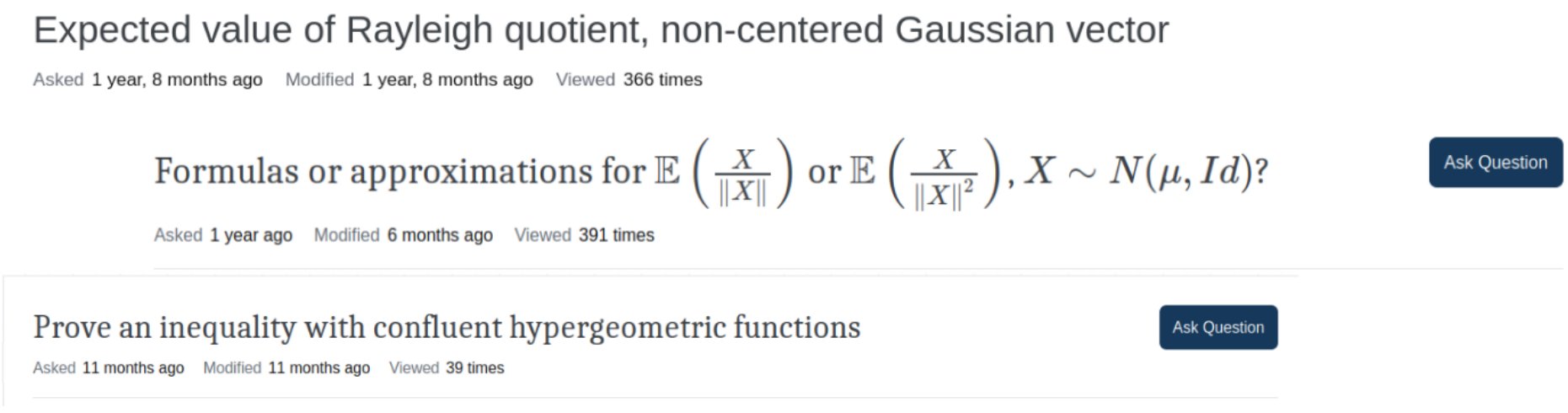
Some amazing tools that helped me

- How does a biologist get all this math done?
- Online math forums

Some amazing tools that helped me

Math forums:





Some amazing tools that helped me

Math forums:

- Using math/programming/statistics forums takes work
- They can help you at any level of knowledge
- Experts around the world happy to help

Conclusion

- Neural variability and divisive normalization interact
- We have a model that captures this interaction and provides us with insight
- The model can be fit to data and used to make predictions

Thanks:

Thanks:

jmarkov
784 4 11

whuber ♦
332k 63 786 1.3k

mathlove
146k 9 121 297

Ben
854 4 4

– user225256

Roland F
3,914 1 2 13

whpowell96
7,408 1 11 23